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Research Article

COMPUTATION CHEZY'S COEFFICIENT IN A SEMI-ELLIPTICAL PIPE

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ABSTRACT

In the hydraulic field, the Chezy's flow resistance coefficient in canals and pipes is often chosen arbitrarily. This value is tabulated independently of the depth of the flow or hydraulic radius and even less of the Reynolds number. This coefficient is usually influenced by a number of parameters that must be defined and considered. The objective of this study is to examine, on the one hand, the variation of the Chezy's flow resistance coefficient for a semi-elliptical pipe under the hypothesis of an uniform flow with free surface, and to determine on the other hand, the expressions which govern them. Further, it must search expression for Chezy's with consideration of the geometric characteristics of the pipe and hydraulic flow. This study is based on the rough model method (RMM).

Keywords: (Semi-Elliptical Pipe, Chezy's Coefficient, Uniform flow, Rough Model Method.)

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1. INTRODUCTION

In the eighteenth century, Antoine Chezy designed a canal on the river Yvette near Paris [1]. From experiments on the Courpalet canal and the Seine, Chezy developed what is now known as the Chezy's formula for comparing uniform flows in open channels [2]. This relationship highlights the link function between depth and mean velocity. These two factors are the most important in the flow, so hydraulic researchers give them a lot of importance. The functional relationship between the mean velocity and the mean depth of a free water surface is determined by the total resistance to flow [3]. This resistance is parameterized by the Chezy's coefficient of resistance that often edit by the letter "C".

In practice, there are tables which give the values of the coefficient C depending on the nature of the materials constituting the internal walls of the channels and pipes [4]. However, this approach does not correspond to the physical reality of the flow, because the coefficient of resistance depends in mainly on the geometry of the pipe and the hydraulic characteristics of the flow.

This is confirmed by several researchers in the field, where the Chezy's coefficient is constantly variable according to multiple parameters. The behavior of C can be inferred directly from that of friction factor [5]. The cross-sectional shape of the channel and differences in bed and bank roughness has a substantial effect on flow resistance[6]. Chezy's roughness coefficient is not constant but it varies in a wide range [7]. On besides, the normal depth of water has an impact on the Chezy's coefficient *C* [1]. According to the last testimonies and based on fundamental equations in hydraulics, we will elaborate in this research work, simple and explicit equations for the calculation of the Chezy's coefficient then we study the variations of this coefficient with consideration of different geometric characteristics of the pipe and hydraulic flow. This study would be done in a semi-elliptical pipe. Closed non circular sections are frequently employed for sewers carrying large discharges [8]. In our study we base on the rough model method RMM [9]. An example of an application will be proposed in order to better appreciate the simplicity, speed and efficiency of the proposed relations.

2. GEOMETRIC CHARACTERISTICS

Figure 1 shows four geometric spaces that the flow [8] can occupy, depending on the value of the filling rate $\eta = y_n / D$. Where η is the filling rate and D is the vertical diameter.

- $0 \le \eta \le 0.09605$,the flow width $\overleftrightarrow{ab}$, is located in the lowest circular part of the conduct.
- $0.09605 \le \eta \le 5/24$, the flow width $\overleftrightarrow{de}$, is located in the space delimited by the arcs of circle $\widehat{EA}$ et $\widehat{DB}$.

- $5/24 \le \eta \le 0.85441$, the flow width $\overleftrightarrow{gf}$, is located in the space delimited by the arcs of circle $\widehat{GE}$ et $\widehat{FD}$.
- $0.85441 \le \eta \le 1$, the flow width $\overleftrightarrow{ij}$, is located in the highest circular part of the conduct.

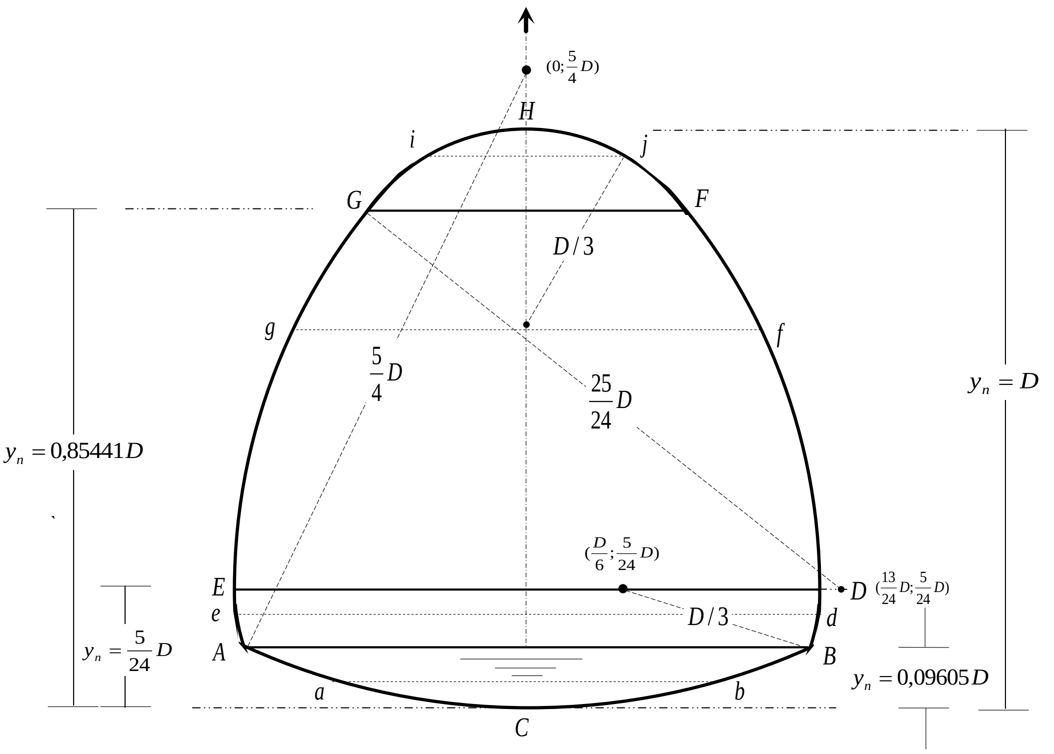


Fig.1. Schematic definition of semi-elliptical conduct

3. GEOMETRIC AND HYDRAULIC PROPERTIES

The characteristics of the flow, the wetted perimeter P' the water area A and the hydraulic radius R_h depend on the filling rate $\eta = y_n / D$ where y_n is the normal depth of the flow. In addition, these characteristics are expressed by different relationships depending on the geometric location of the flow.

i. $0 \le \eta \le 0.09605$

- $P = D\sigma(\eta)$ (1)

The function $\sigma(\eta)$ is defined as follow:

$$\sigma(\eta) = 2.5\cos^{-1}(1 - 0.8\eta) \quad (2)$$

- $A = D^2\varphi(\eta)$ (3)

 Where the function $\varphi(\eta)$ is by definition:

$$\varphi(\eta) = 1.5625\left[\cos^{-1}(1-0.8\eta) - 2(1-0.8\eta)\sqrt{0.4\eta(1-0.4\eta)}\right] \quad (4)$$

- $R_h = D\dfrac{\varphi(\eta)}{\sigma(\eta)}$ (5)

ii. $0.09605 \le \eta \le 5/24$

- $P = D\zeta(\eta)$ (6)

 The function $\zeta(\eta)$ is defined as follow:

$$\zeta(\eta) = 1{,}21548 - \frac{2}{3}\sin^{-1}(0{,}625 - 3\eta) \quad (7)$$

- $A = D^2\vartheta(\eta)$ (8)

 Where the function $\vartheta(\eta)$ is by definition:

$$\vartheta(\eta) = 0{,}103428 + \frac{\eta}{3} - \left(\frac{1}{9}\right)\sin^{-1}(0{,}625 - 3\eta) - \left(\frac{1}{9}\right)(0{,}625 - 3\eta)\sqrt{1-(0{,}625-3\eta)^2} \quad (9)$$

- $R_h = D\dfrac{\vartheta(\eta)}{\zeta(\eta)}$ (10)

iii. $5/24 \le \eta \le 0.85441$

- $P = D\lambda(\eta)$ (11)

 The function $\lambda(\eta)$ is defined as follow:

$$\lambda(\eta) = 1{,}21548 + \left(\frac{25}{12}\right)\sin^{-1}\left(\frac{24}{25}\eta - \frac{1}{5}\right) \quad (12)$$

- $A = D^2\tau(\eta)$ (13)

 Where the function $\tau(\eta)$ is by definition:

$$\tau(\eta)=0,39856-\frac{13}{12}\eta+\frac{625}{576}\left[\sin^{-1}\left(\frac{24}{25}\eta-\frac{1}{5}\right)+\left(\frac{24}{25}\eta-\frac{1}{5}\right)\sqrt{1-\left(\frac{24}{25}\eta-\frac{1}{5}\right)^2}\right] \quad (14)$$

- $R_h=D\dfrac{\tau(\eta)}{\lambda(\eta)}$ (15)

iv. $0.85441\leq\eta\leq1$

- $P=D\chi(\eta)$ (16)

The the function $\chi(\eta)$ is defined by:

$$\chi(\eta)=3.25674-\frac{2}{3}\cos^{-1}(3\eta-2) \quad (17)$$

- $A=D^2\psi(\eta)$ (18)

Where the function $\psi(\eta)$ is by definition:

$$\psi(\eta)=0,78315-\frac{1}{9}\left[\sin^{-1}\sqrt{1-(3\eta-2)^2}-\sqrt{1-(3\eta-2)^2}\,(3\eta-2)\right] \quad (19)$$

- $R_h=D\dfrac{\psi(\eta)}{\chi(\eta)}$ (20)

4. THE GENERAL RELATIONSHIP OF THE CHEZY'S RESISTANCE COEFFICIENT

The discharge flows Q by a conduct of any shape is expressed by the following relation of Achour and Bedjaoui [10], either:

$$Q=-4\sqrt{2g}\,A\sqrt{R_hS}\,\log\left(\frac{\varepsilon}{14,8R_h}+\frac{10.04}{R_e}\right) \quad (21)$$

Where S is the slope of the conduct, R_e is a Reynolds number, ε is the absolute roughness and g is the acceleration due to gravity. The Reynolds number R_e is governed by the following equation [10]:

$$R_e = 32\sqrt{2}\,\frac{\sqrt{gSR_h^3}}{\nu} \quad (22)$$

Where ν is the kinematic viscosity.

The hydraulic radius $R_{h.f}$ in the full state corresponding to the filling rate $\eta = 1$ is, by virtue of the relation (20):

$$R_{h,f} \cong 0.24047\,D \quad (23)$$

Therefore, the Reynolds number $R_{e.f}$ in the full state of the conduct is according to relation (22):

$$R_{e.f} \cong 5{,}33671\,\frac{\sqrt{gSD^3}}{\nu} \quad (24)$$

On the other hand, the relation of Chezy's is defined by following formula:

$$Q = CA\sqrt{R_h S} \quad (25)$$

Through a comparison of the relations (25) and (21), we can deduce that the Chezy's flow resistance coefficient " C " is such as:

$$C = -4\sqrt{2g}\,\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R_e}\right) \quad (26)$$

Or, in dimensionless form:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}\,\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R_e}\right) \quad (27)$$

By using the relationships (5), (10), (15) and (20), we can write for:

i. $0 \le \eta \le 0.09605$

The relation (22) leads to: $R_e = 8{,}47990\left[\frac{\varphi(\eta)}{\sigma(\eta)}\right]^{3/2} R_{e.f}$ (28)

The relation (27) leads to: $\frac{C}{\sqrt{g}} = -4\sqrt{2}\,\log\left(\frac{\varepsilon/D}{14.8[\varphi(\eta)/\sigma(\eta)]} + \frac{1.18397}{R_{e.f}[\varphi(\eta)/\sigma(\eta)]^{3/2}}\right)$ (29)

ii. $0.09605 \le \eta \le 5/24$

The relation (22) is written: $R_e = 8.47990\left[\frac{\vartheta(\eta)}{\zeta(\eta)}\right]^{3/2} R_{e.f}$ (30)

The relation (27) is written: $\frac{C}{\sqrt{g}} = -4\sqrt{2}\log\left(\frac{\varepsilon/D}{14.8[\vartheta(\eta)/\zeta(\eta)]} + \frac{1.18397}{R_{e.f}[\vartheta(\eta)/\zeta(\eta)]^{3/2}}\right)$ (31)

iii. $5/24 \le \eta \le 0.85441$

The relation (22) leads to: $R_e = 8.47990\left[\frac{\tau(\eta)}{\lambda(\eta)}\right]^{3/2} R_{e.f}$ (32)

The relation (27) leads to: $\frac{C}{\sqrt{g}} = -4\sqrt{2}\log\left(\frac{\varepsilon/D}{14.8[\tau(\eta)/\lambda(\eta)]} + \frac{1.18397}{R_{e.f}[\tau(\eta)/\lambda(\eta)]^{3/2}}\right)$ (33)

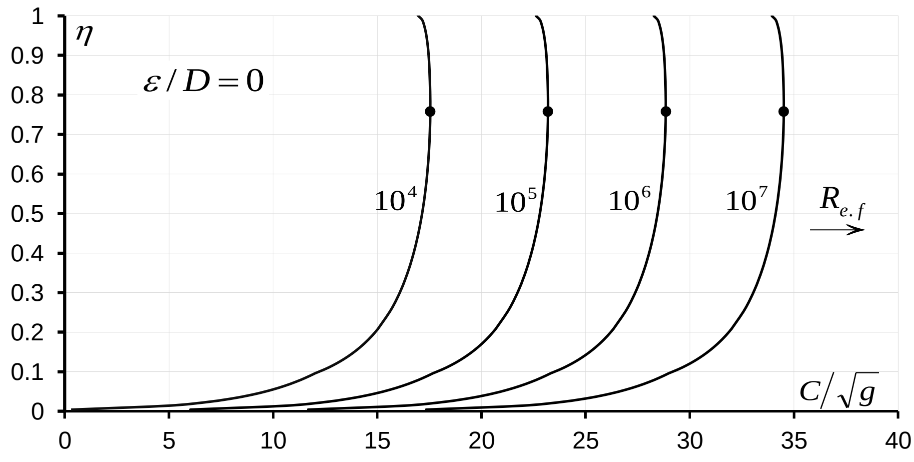
iv. $0.85441 \le \eta \le 1$

The relation (22) leads to: $R_e = 8.47990\left[\frac{\psi(\eta)}{\chi(\eta)}\right]^{3/2} R_{e.f}$ (34)

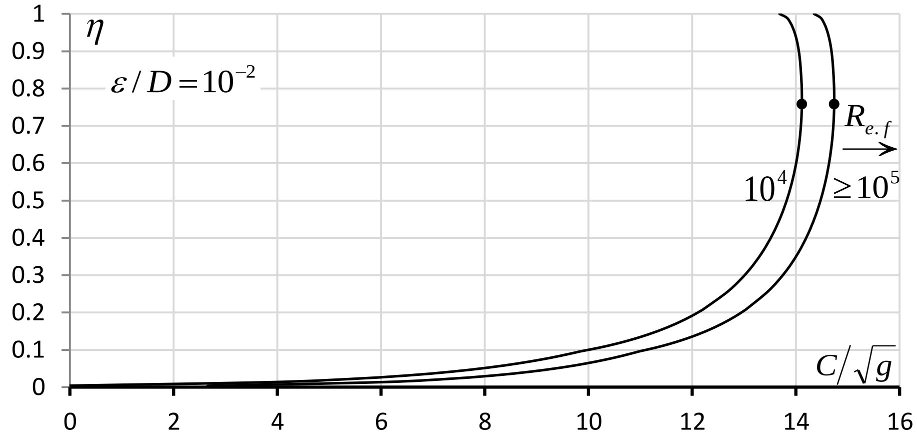
The relation (27) leads to: $\frac{C}{\sqrt{g}} = -4\sqrt{2}\log\left(\frac{\varepsilon/D}{14.8[\psi(\eta)/\chi(\eta)]} + \frac{1.18397}{R_{e.f}[\psi(\eta)/\chi(\eta)]^{3/2}}\right)$ (35)

5. VAIRATION OF CHEZY'S RESISTANCE COEFFICIENT

According to the relations (29), (31), (33) and (35), dimensionless parameter have been represented graphically in figure (2) for the extreme relative roughness $\varepsilon/D = 0$ and $\varepsilon/D = 0,01$.



a)



b)

Fig.2. Variation of $C/\sqrt{g}$ versus η and $R_{e.f}$ according to Eq. (29), (31), (33) et (35), for the extreme relative roughness, (**a**)**:** $\varepsilon / D = 0$; (**b**) : $\varepsilon / D = 0.01$.
(●) Maximum value $C_{\max}/\sqrt{g}$ $\eta \cong 0.77212$

For the practical values of η such as $\eta \geq 0.2$, it appears from figure 2 (a and b) the following observations:

It appears from figure (2) (a and b) that for the same value of the Reynolds number $R_{e.f}$ and whatever the relative roughness ε / D, $C/\sqrt{g}$ increases with the increase of the filling rate η . This increase is very fast in the approximate range $0 \leq \eta \leq 0,2$, while it is slower beyond $\eta = 0,2$. For all the considered relative roughness, the curves show that $C/\sqrt{g}$ reaches a maximum for the value $\eta \cong 0,77212$. Finally, the figure shows that the variation curves of $C/\sqrt{g}$ are gradually tightening as the increases of relative roughness ε / D. We can see in figure 2.b, corresponding to the highest considered relative roughness, that the curves of variation of $C/\sqrt{g}$ are extremely close to each other and merge beyond the Reynolds number. This corresponds to the rough turbulent regime for which $C/\sqrt{g}$ is independent of the Reynolds number $R_{e.f}$ and therefore the kinematic viscosity of the flowing liquid.

6. COMPUTATION OF CHEZY’S RESISTANCE COEFFICIENT

6.1. THE ROUGH MODEL METHOD (RMM)

The rough reference model [9] that we consider is in fact a semi-elliptical conduct, characterized by the height D, and relative roughness $\overline{\varepsilon}/\overline{D} = 0{,}037$ arbitrarily chosen. The symbol "$\overline{}$" is assigned to all the hydraulic parameters of the flow and geometric of the reference conduct. This conduit is the seat of a supposed flow located in the rough turbulent domain and friction factor is then calculated according to the Colebrook-white equation [11], for $R_e \to \infty$, is:

$$\overline{f} = \left[-2\log\left(\frac{\overline{\varepsilon}/\overline{D}}{3.7}\right)\right]^{-2} = \left[-2\log\left(\frac{0.037}{3.7}\right)\right]^{-2} = \frac{1}{16}$$

The value $\overline{f} = 1/16$, can translate in a Chezy's resistance coefficient:

$$\overline{C} = \sqrt{8g/\overline{f}} = 8\sqrt{2g} = \text{Constant} \qquad (36)$$

The (RMM) states that any linear dimension *"L"* of a conduct and the homologous linear dimension of the rough model $\overline{L}$ are related by the following relation [12], applicable to the whole turbulent domain:

$$L = \psi \overline{L} \quad (37)$$

Where ψ is a non-dimensional correction factor of linear dimension, and which is governed by the following explicit equation [13]:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon}{19\overline{R}_h} + \frac{8.5}{\overline{R}_e}\right)\right]^{-2/5} \qquad (38)$$

In this relation, $\overline{R_h}$ and $\overline{R}_e$ are respectively designate the hydraulic radius and the Reynolds number in the rough model.

According to the RMM, Chézy's resistance coefficient to flow is defined by the relation:

$$C = \frac{\overline{C}}{\psi^{5/2}} \qquad (39)$$

Taking into account the relationship (36):

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} \qquad (40)$$

6.2. NON-DIMENSIONAL CORRECTION FACTOR

Depending on the range of values of the filling rate η , the hydraulic radius $\overline{R_h}$ of rough model , is defined by similar relations to relations (5), (10), (15) and (20), as well as the Reynolds number $\overline{R_e}$ is given by one of the relations (28), (30), (32) and (34). So:

i. $0 \le \eta \le 0.09605$

$$\overline{R_h} = \overline{D}\,\frac{\varphi(\eta)}{\sigma(\eta)} \quad (41)$$

$$\overline{R}_e = 8.4799\left[\frac{\varphi(\eta)}{\sigma(\eta)}\right]^{3/2}\overline{R}_{e.f} \quad (42)$$

Inserting Eq. (41) and Eq. (42) into Eq. (38), leads to:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\varphi(\eta)/\sigma(\eta)]} + \frac{1.0023}{[\varphi(\eta)/\sigma(\eta)]^{3/2}\,\overline{R}_{e.f}}\right)\right]^{-2/5} \quad (43)$$

Combining Eq. (40) and Eq. (43), one can write:

$$C = -5.343\sqrt{g}\,\log\left(\frac{\varepsilon/\overline{D}}{19[\varphi(\eta)/\sigma(\eta)]} + \frac{1.0023}{[\varphi(\eta)/\sigma(\eta)]^{3/2}\,\overline{R}_{e.f}}\right) \quad (44)$$

ii. $0.09605 \le \eta \le 5/24$

$$\overline{R_h} = \overline{D}\,\frac{\vartheta(\eta)}{\zeta(\eta)} \quad (45)$$

$$\overline{R}_e = 8.4799\left[\frac{\vartheta(\eta)}{\zeta(\eta)}\right]^{3/2}\overline{R}_{e.f} \quad (46)$$

Inserting Eq. (45) and Eq. (46) into Eq. (38), leads to:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\vartheta(\eta)/\zeta(\eta)]} + \frac{1.0023}{[\vartheta(\eta)/\zeta(\eta)]^{3/2}\,\overline{R}_{e.f}}\right)\right]^{-2/5} \quad (47)$$

Combining Eq. (40) and Eq. (47), one can write:

$$C = -5{,}343\sqrt{g}\,\log\left(\frac{\varepsilon/\overline{D}}{19[\vartheta(\eta)/\zeta(\eta)]} + \frac{1{,}0023}{[\vartheta(\eta)/\zeta(\eta)]^{3/2}\,\overline{R}_{e.f}}\right) \quad (48)$$

iii. $5/24 \le \eta \le 0.85441$

$$\overline{R_h} = \overline{D}\frac{\tau(\eta)}{\lambda(\eta)} \quad (49)$$

$$\overline{R}_e = 8.4799\left[\frac{\tau(\eta)}{\lambda(\eta)}\right]^{3/2}\overline{R}_{e.f} \quad (50)$$

Inserting Eq. (49) and Eq. (50) into Eq. (38), leads to:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\tau(\eta)/\lambda(\eta)]} + \frac{1.0023}{[\tau(\eta)/\lambda(\eta)]^{3/2}\overline{R}_{e.f}}\right)\right]^{-2/5} \quad (51)$$

Combining Eq. (40) and Eq. (51), one can write:

$$C = -5.343\sqrt{g}\log\left(\frac{\varepsilon/\overline{D}}{19[\tau(\eta)/\lambda(\eta)]} + \frac{1.0023}{[\tau(\eta)/\lambda(\eta)]^{3/2}\overline{R}_{e.f}}\right) \quad (52)$$

iv. $0.85441 \le \eta \le 1$

$$\overline{R_h} = \overline{D}\frac{\psi(\eta)}{\chi(\eta)} \quad (53)$$

$$\overline{R}_e = 8.4799\left[\frac{\psi(\eta)}{\chi(\eta)}\right]^{3/2}\overline{R}_{e.f} \quad (54)$$

Inserting Eq. (53) and Eq. (54) into Eq. (38), leads to:

$$\psi = 1,35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\psi(\eta)/\chi(\eta)]} + \frac{1,0023}{[\psi(\eta)/\chi(\eta)]^{3/2}\overline{R}_{e.f}}\right)\right]^{-2/5} \quad (55)$$

Combining Eq. (40) and Eq. (55), one can write:

$$C = -5,343\sqrt{g}\log\left(\frac{\varepsilon/\overline{D}}{19[\psi(\eta)/\chi(\eta)]} + \frac{1,0023}{[\psi(\eta)/\chi(\eta)]^{3/2}\overline{R}_{e.f}}\right) \quad (56)$$

In relations (44), (48), (3), and (52), the *Reynolds numbers* $\overline{R}_{e.f}$ In the rough model the full state is expressed by similar relation to relation (24), is:

$$\overline{R}_{e.f} \cong 5,33671\frac{\sqrt{gS\overline{D}^3}}{\nu} \quad (57)$$

The relation (29), (31), (33) and (35) permits to evaluate the coefficient of Chézy's only if the diameter D of the semi-elliptic conduit is a given of the problem. In the case where D is unknown, it is still possible to calculate the C value, provided to use the rough model method (RMM). This is one of the advantages of this method. The application of the RMM in Chézy's relationship allows the determination of the diameter $\overline{D}$, we admit the following conditions:

$i)\ \overline{Q} = Q;\quad ii)\ \overline{S} = S;\quad iii)\ \overline{\eta} = \eta;\quad iv).\ \overline{\nu} = \nu$

Taking into account conditions (i) and (ii), the discharge $\overline{Q}$ passed by the rough reference model and written as follows:

$$\overline{Q} = Q = \overline{C}\,\overline{A}\sqrt{\overline{R_h}S} \quad (58)$$

In equation (58), the water area $\overline{A}$ is calculated by similar relations to that (3), (8), (13) and (18), and the hydraulic radius $\overline{R_h}$ is given by relations (41), (45), (49) and (53), depending on the range of variation of the filling rate. So:

i. $0 \le \eta \le 0.09605$

The water area of the rough model is:

$$\overline{A} = \overline{D}^2\varphi(\eta) \quad (59)$$

Taking into account Eq. (41), Eq. (36) and Eq. (59), Eq. (58) is written:

$$Q = \left(8\sqrt{2g}\right) \times \left[\overline{D}^2\varphi(\eta)\right] \times \sqrt{\left[\overline{D}\,\frac{\varphi(\eta)}{\sigma(\eta)}\right] \times S}$$

After the calculation, one obtains:

$$\overline{D} = \frac{[\sigma(\eta)]^{0.2}}{2.639[\varphi(\eta)]^{0.6}}\left(\frac{Q}{\sqrt{gS}}\right)^{0.4} \quad (60)$$

ii. $0.09605 \le \eta \le 5/24$

$$\overline{A} = \overline{D}^2\vartheta(\eta) \quad (61)$$

Taking into account Eq. (45), Eq. (36) and Eq. (61), Eq. (58) leads to:

$$\overline{D} = \frac{[\zeta(\eta)]^{0.2}}{2.639[\vartheta(\eta)]^{0.6}}\left(\frac{Q}{\sqrt{gS}}\right)^{0.4} \quad (62)$$

iii. $5/24 \le \eta \le 0.85441$

$\overline{A} = \overline{D}^2 \tau(\eta)$(63)

By introducing the relation (49), (36) and (63) in the relation (58),we can deduce that diameter $\overline{D}$ is such that:

$$\overline{D} = \frac{[\lambda(\eta)]^{0.2}}{2.639[\tau(\eta)]^{0.6}}\left(\frac{Q}{\sqrt{gS}}\right)^{0.4} \quad (64)$$

iv. $0.85441 \le \eta \le 1$

$\overline{A} = \overline{D}^2 \psi(\eta)$(65)

Taking into account Eq. (53), Eq. (36) and Eq. (65), Eq. (58) leads to:

$$\overline{D} = \frac{[\chi(\eta)]^{0.2}}{2.639[\psi(\eta)]^{0.6}}\left(\frac{Q}{\sqrt{gS}}\right)^{0.4} \quad (66)$$

To evaluate the resistance coefficient of Chézy's to flow C, for the known values of the parameters Q, η, ε, S and ν ,the following steps are recommended:

a) With the given values of the discharge Q, the slope of the conduct S and that of the filling rate η, one of the relations (60), (62), (64) or (66) allows to evaluate the diameter of the rough model of reference $\overline{D}$.

b) The known parameters $\overline{D}$, S and ν are introduced in the relation (57) for the calculation of the Reynolds number $\overline{R}_{e.f}$ in the full state.

c) Determining the value of the relative roughness of the model $\varepsilon/\overline{D}$.

d) The coefficient ψ can then be evaluated by one of the relationships (43), (47), (51) or (55).

e) Finally, the resistance coefficient of Chézy's C can be explicitly evaluated by one of the relations (44), (48), (52) or (56) according to the range of values of the filling rate.

7. APPLICATION EXAMPLE

Calculate the Chézy's resistance coefficient C of a semi-elliptical conduct shown in figure (1). For the following data:

$\eta = 0.75$; $Q = 3.15 m^3/s$; $\nu = 10^{-6} m^2/s$; $S = 4.10^{-4}$; $\varepsilon = 0{,}0002\, m$

Solution

1. The value of η belongs to the interval $5/24 \le \eta \le 0.85441$, so the diameter of the referential rough model $\overline{D}$ is given by relation (64). The functions $\lambda(\eta)$ and $\tau(\eta)$ are given by the relations (12) and (14) respectively and take for values:

- $\lambda(0.75) = 1.21548 + \left(\frac{25}{12}\right)\sin^{-1}\left[\left(\frac{24}{25}\times 0.75\right) - \frac{1}{5}\right] = 2.354753$

$$\tau(0.75) = 0.39856 - \frac{13}{12}\times 0.75 + \frac{625}{576}\left[\sin^{-1}\left(\frac{24}{25}\times 0.75 - \frac{1}{5}\right) + \left(\frac{24}{25}\times 0.75 - \frac{1}{5}\right)\sqrt{1 - \left(\frac{24}{25}\times 0.75 - \frac{1}{5}\right)^2}\right] = 0.661383$$

2. The diameter $\overline{D}$ is calculated by the relation (64), is:

$$\overline{D} = \frac{[\lambda(\eta)]^{0.2}}{2.639[\tau(\eta)]^{0.6}}\left(\frac{Q}{\sqrt{gS}}\right)^{0.4} = \frac{[2.354753]^{0.2}}{2.639[0.661383]^{0.6}}\left(\frac{3.15}{\sqrt{9.81\times 0.0004}}\right)^{0.4} = 2.7622081 m$$

3. The relation (57) is applied to determine the number of Reynolds $\overline{R}_{e.f}$ in the full state:

$$\overline{R}_{ef} \cong 5.33671\frac{\sqrt{gi\overline{D}^3}}{\nu} = 5.33671\frac{\sqrt{9.81\times(4.10^{-4})\times(2.7622081)^3}}{10^{-6}} = 1534697.184$$

4. The factor ψ is given by the relation (51):

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\tau(\eta)/\lambda(\eta)]} + \frac{1.0023}{[\tau(\eta)/\lambda(\eta)]^{3/2}\overline{R}_{e.f}}\right)\right]^{-2/5}$$

$$= 1.35\times\left\{-\log\left[\frac{0.0002/2.7622081}{19\times\left(\frac{0.661382706}{2.354752814}\right)} + \frac{1.0023}{\left(\frac{0.661382706}{2.354752814}\right)^{3/2}\times 1534697.184}\right]\right\}^{-2/5} = 0.72411873$$

5. The Chézy's resistance coefficient C is, according to the relation (40) :

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} = \frac{8\times\sqrt{2\times 9{,}81}}{0.7251318^{5/2}} = 79.4172847\ m^{0{,}5}/s$$

8. CONCLUSION

The general relationship of the Chezy's resistance coefficient C was identified by using the the discharge relationship proposed by Achour and Bedjaoui (2006). Chezy's resistance coefficient C is a function of both the relative roughness ε / D, the filling rate η and the Reynolds number $R_{e.f}$, which is itself a function of the slope of the conduct S, the diameter D, and the η, kinematic viscosity ν of fluid flow. So we wrote the functional relationship $f(C, \eta, S, \varepsilon, \nu) = 0$. Depending on the range of values of the filling rate η, we have determined the non dimensional parameter $C / \sqrt{g}$. The relation obtained demonstrates that all the parameters influencing the flow are taken into account, notably the kinematic viscosity ν. It appear according to the filling rate η, the relative roughness ε / D and the Reynolds number $R_{e.f}$ in the full state. The graphical representation of the parameter $C / \sqrt{g}$ demonstrated that the Reynolds number $R_{e.f}$ in the full state plays an important role. The obtained graphs show that $C / \sqrt{g}$ goes through a maximum for $\eta \cong 0{,}77212$. When the diameter D of the conduit is not a data of the problem, the calculation of the Chezy's resistance coefficient could be possible by using the rough model method (RMM).

The results we presented show the effectiveness and use of the RMM method, and especially its applications in the various hydraulic structure and in particular the researches that are conducted in the channels.

This type of method is very useful to avoid calculation errors or failures that may arise during the realization of multiple hydraulic works including in the channels or conducts.

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